

# Practical LDPC coded modulation schemes for the fading broadcast channel with confidential messages

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**Abstract**—The broadcast channel with confidential messages is a well studied scenario from the theoretical standpoint, but there is still lack of practical schemes able to achieve some fixed level of reliability and security over such a channel. In this paper, we consider a quasi-static fading channel in which both public and private messages must be sent from the transmitter to the receivers, and we aim at designing suitable coding and modulation schemes to achieve such a target. For this purpose, we adopt the error rate as a metric, by considering that reliability (security) is achieved when a sufficiently low (high) error rate is experienced at the receiving side. We show that some conditions exist on the system feasibility, and that some outage probability must be tolerated to cope with the fading nature of the channel. The proposed solution exploits low-density parity-check codes with unequal error protection, which are able to guarantee two different levels of protection against noise for the public and the private information, in conjunction with different modulation schemes for the public and the private message bits.

**Index Terms**—Broadcast channel with confidential messages, low-density parity-check codes, physical layer security, quasi-static fading channel, unequal error protection.

## I. INTRODUCTION

One of the basic transmission models for physical layer security is the broadcast channel with confidential messages (BCC) [1]. In this model, there is one transmitter (Alice) who sends both broadcast and confidential information over the channel. The authorized receiver (Bob) is able to decode the whole information, while the non-authorized receiver (Eve) can only have access to the public information, but she should be unable to obtain the confidential information. Bob's and Eve's channels are generally different one each other. A practical context in which the BCC model can be applied is the integration of multiple services at the physical layer. For example, a wireless network could provide a free broadcast service to all users, and also exploit the same channel to provide another service which is restricted to a subset of users.

The BCC model has been extensively studied from the information theory standpoint, mostly with the aim of computing the secrecy capacity regions. This has been done for Gaussian variants of the BCC [2], [3] and also by considering the case of fading channels [4]–[8]. More recently, the secrecy capacity regions have been studied for the BCC with multiple-input

multiple-output (MIMO) [9]–[11] and cooperative communications [12]. In many of these works, coding is considered as an important tool for achieving the reliability and security targets over the BCC, but the abstract model of random coding is often considered [13], and the design of practical coding schemes is not addressed. At the authors' best knowledge, only one proposal of using polar codes as practical codes for transmissions over the discrete memoryless BCC has very recently appeared [14], while no practical solution exists for continuous-output BCCs.

In this work, we consider another important class of powerful error correcting codes, namely low-density parity-check (LDPC) codes, and propose a practical scheme to achieve reliable and secure transmission of public and private information over the BCC. In order to consider a practically meaningful scenario, our analysis is focused on the case of quasi-static fading channel (QSFC) for both Bob and Eve. Following some previous literature [15], [16], the reliability and security performance is measured on the basis of the decoding error probabilities experienced at the receiving side. This way, practical coding schemes can be easily assessed and compared, as we have already done for the Gaussian wire-tap channel [17]–[19].

We show that a transmission scheme able to provide two different levels of protection against noise is needed to achieve the transmission reliability and security targets over the BCC. For this reason, we use some LDPC codes having unequal error protection (UEP) [20], [21]. We consider different modulation formats for the private information bits, and we show that high order modulations are needed.

The organization of the paper is as follows: in Section II we define the system model and the metrics we use. In Section III we address the system feasibility and compute the outage probability for Bob and Eve. In Section IV we describe the UEP LDPC codes we propose to use in this context. In Section V we provide and discuss some numerical examples, and Section VI concludes the paper.

## II. SYSTEM MODEL AND METRICS

The channel model we consider is shown in Fig. 1. Both Bob's and Eve's channels are Rayleigh fading, with fading coefficients  $h_B$  and  $h_E$ , respectively, and also affected by

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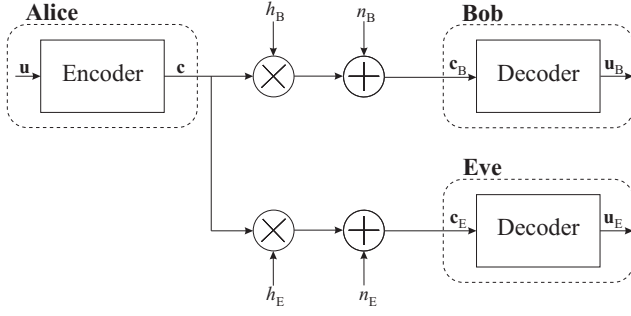


Fig. 1. Fading wire-tap channel model.

additive white Gaussian noise (AWGN),  $n_B$  and  $n_E$ . We suppose that Bob's and Eve's channels are QSFCs, that is, their fading coefficients do not vary during the transmission of each codeword, while they can be modeled as Rayleigh random variables over different codewords. The signal-to-noise ratios (SNRs) of Bob's and Eve's channels are usually different. Therefore, the two vectors received by Bob and Eve,  $\mathbf{c}_B$  and  $\mathbf{c}_E$ , are also different, as well as the two messages they get after decoding, noted by  $\mathbf{u}_B$  and  $\mathbf{u}_E$ , respectively. Bob is an authorized receiver, able to decode the whole information. Eve instead is a non-authorized receiver, able to get only the public message information, whereas she should be unable to obtain any useful information on the secret message.

Each transmitted message is formed by  $n$  bits and includes a public and a confidential part. Since we use error correcting coding, each transmitted message contains  $k$  information bits and  $r = n - k$  redundancy bits. The overall code rate is  $R = \frac{k}{n}$ , and  $R$  also coincides with the overall information rate, expressed in bits per channel use, when we use binary phase shift keying (BPSK) modulation. The transmitted information bits can be divided into a block of  $k_s \leq k$  secret information bits, and another block of  $k_p = k - k_s$  public information bits.

The SNRs on the two channels, noted by  $\gamma^{(B)}$  and  $\gamma^{(E)}$ , result from the combination of the AWGN contribution and the Rayleigh fading contribution. The average SNRs are equal to  $\bar{\gamma}^{(B)}$  and  $\bar{\gamma}^{(E)}$  for Bob and Eve, respectively. According to the Rayleigh fading model,  $h_B$  and  $h_E$  are two Rayleigh random variables, whose real and imaginary parts are Gaussian random variables with zero mean and variance  $1/2$ . Therefore,  $|h_B|^2$  and  $|h_E|^2$  are chi-square distributed, with average value  $E(|h_B|^2) = E(|h_E|^2) = 1$ . It follows that the probability density functions of  $\gamma^{(B)}$  and  $\gamma^{(E)}$  are:

$$p_{\gamma^{(B)}}(x) = \frac{1}{\bar{\gamma}^{(B)}} e^{-x/\bar{\gamma}^{(B)}}, \quad x \geq 0 \quad (1a)$$

$$p_{\gamma^{(E)}}(x) = \frac{1}{\bar{\gamma}^{(E)}} e^{-x/\bar{\gamma}^{(E)}}, \quad x \geq 0 \quad (1b)$$

We suppose to have average channel state information (CSI), that is, Alice knows the values of  $\bar{\gamma}^{(B)}$  and  $\bar{\gamma}^{(E)}$ . Several works in the literature assume to have perfect CSI, that is, Alice knows exactly the values of  $\gamma^{(B)}$  and  $\gamma^{(E)}$  for each transmitted codeword. We prefer to make the assumption of having only average CSI, since it is more realistic for a practical system like the one we want to address.

### A. Reliability and security targets

In order to design practical coding and modulation schemes for the considered BCC, we need some metrics which allow to take into account and assess the performance achieved by each specific instance of the system. For this purpose, we adopt the error rate as a metric both for reliability and security.

Let  $P(\gamma)$  denote the overall frame error rate (FER) as a function of the SNR  $\gamma$ . In other terms,  $P(\gamma)$  is the probability that one or more of the  $k$  information bits are in error within a received frame of  $n$  bits. Since each block of  $k$  information bits contains a public and a secret part, we denote by  $P_p(\gamma)$  and  $P_s(\gamma)$  the block error rate (BLER) for each of these two parts, respectively.

As done in some recent literature [15]–[19], we define the security and reliability targets in terms of the decoding error probabilities experienced by Bob and Eve. Given two small threshold values,  $\delta$  and  $\epsilon$ , we define the security and reliability targets as follows:

$$P_p(\gamma^{(B)}) \leq \delta, \quad (2a)$$

$$P_p(\gamma^{(E)}) \leq \delta, \quad (2b)$$

$$P_s(\gamma^{(B)}) \leq \delta, \quad (2c)$$

$$P_s(\gamma^{(E)}) \geq 1 - \epsilon. \quad (2d)$$

Conditions (2a)–(2c) ensure the desired reliability, while (2d) guarantees a sufficiently large error probability on the secret information at Eve's. Having defined the security target in terms of the BLER, one could object that, when a block is in error, this does not necessarily mean that its bits are erred with probability 0.5 (which would be the desired maximum uncertainty condition from the information theory standpoint). Therefore, we cannot state that the system achieves perfect secrecy. However, we can say that the system achieves a looser notion of *weak secrecy*, as defined in [22]. In fact, when Eve's BLER on the secret information is almost 1, we know that she has some uncertainty on the secret information bits. This small amount of uncertainty can be exploited to achieve a desired higher level of security through suitable transformations. For example, an all-or-nothing transform (AONT) [23] can be used to link a set of transmitted blocks together, in such a way that their information can be recovered only when all of them are decoded without errors. Several examples of AONTs can be found in the literature. When transmission occurs over noisy channels, like in this case, we have shown in [17]–[19], [24] that scrambling the information bits through a linear (and dense) map can be sufficient to approach an AONT, thanks to the randomness of the errors induced by the channel.

On the other hand, using the error rate as a reliability and security metric imposes some restrictions on our analysis. First of all, the error rate depends on the decoder. Therefore, we should suppose that Eve uses an optimal decoder to attack the system, that is, a maximum likelihood (ML) decoder for continuous-output channels. However, for sufficiently long LDPC codes, it is known that belief propagation iterative decoders are able to approach the ML decoding performance. Therefore, we can consider the performance achieved by Eve

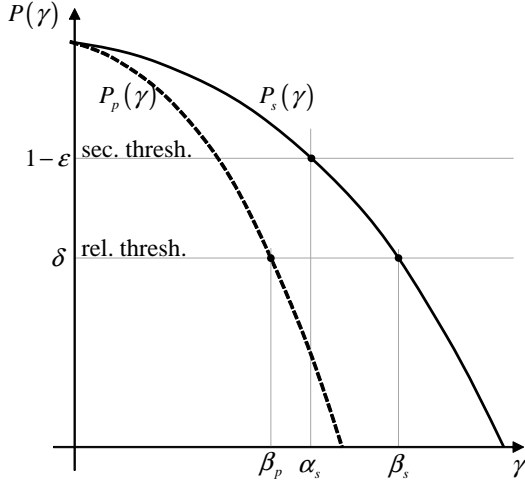


Fig. 2. Expected block error rate curves for the public and secret messages as functions of the SNR.

through iterative decoding as a reliable estimate of the optimal decoder performance. This also avoids the need to consider that Eve uses other decoders, like, for example, those based on ordered statistics decoding (OSD) algorithms [25], which exploit the concept of information set decoding (ISD) [26], aided by the soft information available at the channel output. An algorithm similar to ISD was also proposed in [27] to attack two authentication protocols which exploit some noisy observations. In that case, however, the attacker can also take advantage of the fact that a fixed secret key is used as the starting point to compute the transmitted data.

Finally, we observe that the secrecy condition we define by using the error rate as a metric is also weak in that it may imply to transmit at a secret rate which is smaller than the equivocation rate at Eve's. Therefore, there may be some leakage on the confidential information part which must be compensated for by using higher layer techniques (like AONTs). Estimating the equivocation rate of this system and introducing some modifications (like the use of some intentional randomness) to achieve a secret rate which approaches the equivocation rate will be the object of future works.

### III. SYSTEM FEASIBILITY AND OUTAGE

Let us suppose that we use a coding and modulation scheme which offers a higher level of protection against noise to the public information part with respect to the secret information part. Typical error rate curves for this case are reported in Fig. 2, where  $\beta_p$  and  $\beta_s$  denote the minimum SNRs which are needed to meet the reliability conditions on the public and the secret information, respectively, while  $\alpha_s$  is the maximum SNR which is allowed to meet the security condition on the secret information.

Based on the  $P_p$  and  $P_s$  curves, we can rewrite the conditions (2) in terms of  $\gamma^{(B)}$  and  $\gamma^{(E)}$ . In fact, the conditions (2a) and (2c) are equivalent to set

$$\gamma^{(B)} \geq \max\{\beta_p, \beta_s\} = \beta_s, \quad (3)$$

while the conditions (2b) and (2d) are satisfied if and only if

$$\beta_p \leq \gamma^{(E)} \leq \alpha_s. \quad (4)$$

Since  $1 - \epsilon > \delta$  by definition, it follows that the condition (4) can be met only when the public information is more protected against noise than the secret information, that is the situation depicted in Fig. 2. We observe that, in this context, the condition in which Eve has a degraded channel with respect to Bob does not suffice to make the system feasible as it occurs for the wire-tap channel model. In principle, the system is feasible even when  $\alpha_s = \beta_p$ . In practice, however, we need that  $\alpha_s > \beta_p$  to ensure that the system remains feasible even when  $\gamma^{(E)}$  has some fluctuations, like in the case of fading channels we consider, as we will discuss next.

Provided that the system is feasible, we can assess and compare different coding and modulation schemes by computing the security gap  $S_g$ , which is defined as the ratio between the limit values of Bob's and Eve's SNRs which are needed to meet the reliability and security conditions. Designing coding and modulation schemes which achieve small security gaps is important, since this means that reliability and security can be achieved even with a small degradation of Eve's channel with respect to Bob's channel.

#### A. Bob's outage

When Bob receives a transmitted codeword, he must be able to meet the reliability conditions (2a) and (2c). From (3) we have that both these conditions are met when  $\gamma^{(B)} \geq \beta_s$ , hence an outage event occurs when  $\gamma^{(B)} < \beta_s$ . We denote by  $\eta$  the probability of such an event, and from (1a) we have

$$\begin{aligned} \eta &= P\{0 \leq \gamma^{(B)} < \beta_s\} = \int_0^{\beta_s} p_{\gamma^{(B)}}(x) dx \\ &= 1 - \exp\left(-\frac{\beta_s}{\bar{\gamma}^{(B)}}\right). \end{aligned} \quad (5)$$

We suppose to have average CSI on both channels, hence the transmission power can be chosen such that the probability of outage is not greater than some fixed value  $\eta_{\max}$ , that is:

$$\bar{\gamma}^{(B)} \geq \bar{\gamma}_{\min}^{(B)} = -\frac{\beta_s}{\ln(1 - \eta_{\max})}. \quad (6)$$

#### B. Eve's outage

When Eve receives a transmitted codeword, two outage events can occur:

- The reliability condition (2b) on the public information is not met. We define  $\omega_r$  the probability of this event.
- The security condition (2d) on the secret information is not met. We define  $\omega_s$  the probability of this event.

Based on (1b), we have

$$\begin{aligned} \omega_r &= P\{0 \leq \gamma^{(E)} < \beta_p\} = \int_0^{\beta_p} p_{\gamma^{(E)}}(x) dx \\ &= 1 - \exp\left(-\frac{\beta_p}{\bar{\gamma}^{(E)}}\right) \end{aligned} \quad (7)$$

and

$$\begin{aligned}\omega_s &= P\left\{\gamma^{(E)} > \alpha_s\right\} = \int_{\alpha_s}^{\infty} p_{\gamma^{(E)}}(x)dx \\ &= \exp\left(-\frac{\alpha_s}{\bar{\gamma}^{(E)}}\right).\end{aligned}\quad (8)$$

Since the two outage events are incompatible, the overall outage probability for Eve is

$$\omega = \omega_r + \omega_s = 1 - \exp\left(-\frac{\beta_p}{\bar{\gamma}^{(E)}}\right) + \exp\left(-\frac{\alpha_s}{\bar{\gamma}^{(E)}}\right). \quad (9)$$

As we suppose to have average CSI on both channels, we can assume that  $\bar{\gamma}^{(E)}$  is chosen in such a way that  $\omega$  equals its minimum,  $\omega_{\min}$ . This optimal value of  $\bar{\gamma}^{(E)}$ , named  $\bar{\gamma}_{\text{opt}}^{(E)}$ , can be easily found by computing the derivative of  $\omega$  with respect to  $\bar{\gamma}^{(E)}$ , that is,

$$\frac{d\omega}{d\bar{\gamma}^{(E)}} = \frac{\alpha_s \exp\left(-\frac{\alpha_s}{\bar{\gamma}^{(E)}}\right) - \beta_p \exp\left(-\frac{\beta_p}{\bar{\gamma}^{(E)}}\right)}{(\bar{\gamma}^{(E)})^2}. \quad (10)$$

Then,  $\bar{\gamma}_{\text{opt}}^{(E)}$  is obtained by setting  $\frac{d\omega}{d\bar{\gamma}^{(E)}} = 0$ . This way, we have

$$\bar{\gamma}_{\text{opt}}^{(E)} = \frac{\beta_p - \alpha_s}{\ln\left(\frac{\beta_p}{\alpha_s}\right)}. \quad (11)$$

Therefore, by taking Bob's and Eve's outage probabilities (i.e.,  $\eta_{\max}$  and  $\omega_{\min}$ ) into account, we can compute the security gap as

$$S_g = \frac{\bar{\gamma}_{\min}^{(B)}}{\bar{\gamma}_{\text{opt}}^{(E)}}. \quad (12)$$

#### IV. UEP LDPC CODES FOR THE BCC

In order to achieve the two levels of protection which are requested for the public and the secret information blocks, we use an LDPC code with UEP. We are interested in finding an UEP LDPC code with a length of  $n$  bits, able to achieve two different levels of protection against noise on the set of  $k < n$  information bits. This requirement fits well with several design approaches proposed in the literature [20], [21], [28], which aim at dividing the codeword bits into three protection classes (PCs), named PC1, PC2 and PC3, respectively:

- The PC1 contains  $k_1 < k$  information bits which are the most protected against noise.
- The PC2 contains  $k_2 = k - k_1$  information bits which are less protected against noise than those in PC1.
- The PC3 contains the  $r = n - k$  redundancy bits.

Hence, for the use in the considered scenario, we can design a code with these three PCs, and then map the public information bits into PC1 (i.e.,  $k_p = k_1$ ) and the secret information bits into PC2 (i.e.,  $k_s = k_2$ ).

Codes of this kind can be obtained by designing suitable node degree distributions and then grouping the codeword bits based on their node degrees. More in detail, first of all, the variable node degree distribution must be chosen in such a way as to achieve a good convergence threshold under iterative decoding. To have UEP, instead, the degree distribution must include both very low and rather high variable node degrees.

Since the highest degrees ensure greater protection, the variable nodes with such degrees which correspond to information bits form the PC1. Among the remaining variable nodes, with low degrees, those corresponding to information bits form the PC2. Finally, the variable nodes associated to redundancy bits form the PC3.

The design starts from an optimized variable node degree distribution from the edge perspective, which is expressed as a polynomial,  $\lambda(x) = \sum_{i=1}^{\bar{d}_v} \lambda_i x^{i-1}$ , with real coefficients. The coefficient  $\lambda_i$  coincides with the fraction of edges connected to variable nodes having degree  $i$ , and  $\bar{d}_v$  is the maximum variable node degree. Then,  $\lambda(x)$  is converted from the edge perspective to the node perspective, thus obtaining the polynomial  $\nu(x) = \sum_{i=1}^{\bar{d}_v} \nu_i x^i$ , whose coefficients  $\nu_i$  are related to the  $\lambda_i$ 's as follows:

$$\begin{aligned}\nu_i &= \frac{\lambda_i/i}{\sum_{j=1}^{\bar{d}_v} \lambda_j/j}, \\ \lambda_i &= \frac{\nu_i \cdot i}{\sum_{j=1}^{\bar{d}_v} \nu_j \cdot j}.\end{aligned}\quad (13)$$

The node perspective is useful to assign the variable node bits to the PCs. In particular, the number of bits in PC1, that is the most protected class, is computed by summing the fractions of nodes (i.e., the values of  $\nu_i$ ) corresponding to the highest values of  $i$  (i.e., to the highest variable node degrees).

The same reasoning can be applied to the check nodes degree distributions, by denoting with  $\rho(x)$  and  $c(x)$  the check node degree distributions from the edge and the node perspectives, respectively, and by replacing  $\lambda$  with  $\rho$ ,  $\nu$  with  $c$ , and  $\bar{d}_v$  with  $\bar{d}_c$ , where  $\bar{d}_c$  is the maximum check node degree.

Concerning the design of the check node degree distribution, we adopt a concentrated distribution (i.e., with only two degrees, concentrated around the mean). This solution has the advantage of being very simple, while achieving good performance. This way, we obtain

$$c(x) = ax^{\lfloor c_m \rfloor} + bx^{\lceil c_m \rceil}, \quad (14)$$

where  $c_m = \frac{E}{r} = \frac{\sum_i v_i \cdot j}{(1-R)}$  and  $E$  is the total number of edges in the Tanner graph. The values  $a$  and  $b$  are computed as

$$a = \lceil c_m \rceil - c_m, \quad b = c_m - \lfloor c_m \rfloor. \quad (15)$$

Once having designed the variable and check nodes degree distributions, a practical code with an arbitrary finite length can be obtained by designing its  $r \times n$  parity-check matrix in such a way as to match the two degree distributions. This can be accomplished through several algorithms. Among them, we adopt the *zigzag-random* construction [21], [29].

Since we use these codes to map the first two PCs to the public and the secret information bits, it is advisable to ensure that a high level of separation exists between these two classes, in such a way that possible fluctuations of the error rate on one of them do not affect the error rate on the other. For this purpose, we design the parity-check matrix in such a way as to keep the number of parity-check equations which are common between the first two PCs as small as possible, while still achieving good performance.



TABLE I  
PERFORMANCE OF THE CONSIDERED CODING AND MODULATION  
SCHEMES (ALL VALUES ARE IN dB, EXCEPT THE OUTAGE PROBABILITY)

Scheme	$\alpha_s$	$\omega_{\min}$ ( $\eta_{\max}$ )	$\bar{\gamma}_{\text{opt}}^{(E)}$	$\beta_s$	$\bar{\gamma}_{\min}^{(B)}$	$S_g$
BPSK	2.95	0.81	1.90	5.35	3.14	1.24
64 QAM	12.25	0.24	7.70	14.12	19.73	12.03
128 QAM	15.78	0.13	10.25	17.67	26.23	15.98
512 QAM	20.64	0.05	13.99	22.94	35.84	21.85
2048 QAM	25.27	0.02	17.73	28.49	45.44	27.71

## V. NUMERICAL EXAMPLES

In order to provide some numerical examples, we consider an UEP LDPC code with  $n = 4096$  and overall code rate  $R = 1/2$ . Its variable nodes degree distribution is taken from [20, Table 3], with some minor modifications which are needed to change the proportion between the PC1 and the PC2:

$$\lambda(x) = 0.0025x^{19} + 0.0009x^{18} + 0.0031x^{17} + 0.0630x^{16} + 0.3893x^{15} + 0.2985x^2 + 0.2427x. \quad (16)$$

The corresponding degree distribution from the node perspective is

$$\nu(x) = 0.0005x^{20} + 0.0002x^{19} + 0.0007x^{18} + 0.0151x^{17} + 0.0835x^{16} + 0.4054x^3 + 0.4946x^2. \quad (17)$$

We observe from (17) that the variable nodes can be grouped into two classes with node degrees  $\leq 3$  or  $\geq 16$ . Therefore, the nodes in PC1 will be those having degree  $\geq 16$ , while the others will be in PC2 or PC3, depending on their association to information or redundancy bits. This way, we find that PC1 and PC2 contain, respectively, 20% and 80% of the information bits.

The performance of this code has been assessed by simulating transmission over a Gaussian channel with SNR per bit equal to  $\gamma$ , and by performing decoding through the log-likelihood ratio sum-product algorithm (LLR-SPA). The bits in the PC1 are always transmitted by using BPSK modulation, while for the bits in the PC2 several quadrature amplitude modulation (QAM) formats have also been tested. For the latter, we have adopted the labeling known as Yarg [30], which has been suitably designed for physical layer security contexts. Concerning QAM transmissions, they have been implemented through a pragmatic approach, by mapping groups of bits into QAM symbols, and then using a classical symbol-to-bit soft metric conversion before LDPC decoding. The performance obtained, in terms of  $P_p(\gamma)$  and  $P_s(\gamma)$ , is reported in Fig. 3.

We fix two values for the reliability and security thresholds, that is,  $\delta = 10^{-4}$  and  $\epsilon = 0.1$ . Based on these choices, from Fig. 3 we obtain  $\beta_p = 0.75$  dB, while  $\alpha_s$  and  $\beta_s$  vary according to the modulation scheme used for the secret information bits. The values taken by  $\alpha_s$  and  $\beta_s$  for the considered modulation schemes are reported in Table I.

Starting from the values of  $\alpha_s$  and  $\beta_s$ , we can compute Eve's overall outage probability  $\omega$  (9), as a function of Eve's average SNR per bit,  $\bar{\gamma}^{(E)}$ . The values of  $\omega$ , so obtained, are reported in Fig. 4 for the considered secret information modulation formats. Then, the value of  $\omega_{\min}$  is easily obtained,

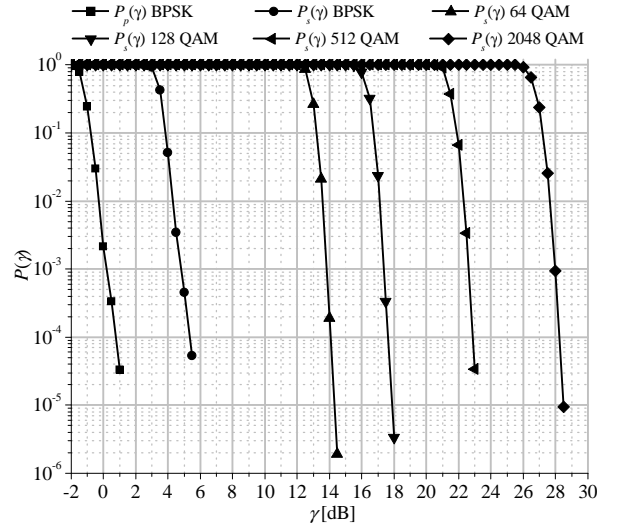


Fig. 3. Error rate curves for an UEP LDPC code with length 4096 and PC1 and PC2 with proportions 20% – 80%. The bits in the PC1 are always BPSK modulated, while the performance of several QAM schemes with Yarg labeling on the bits in the PC2 is reported.

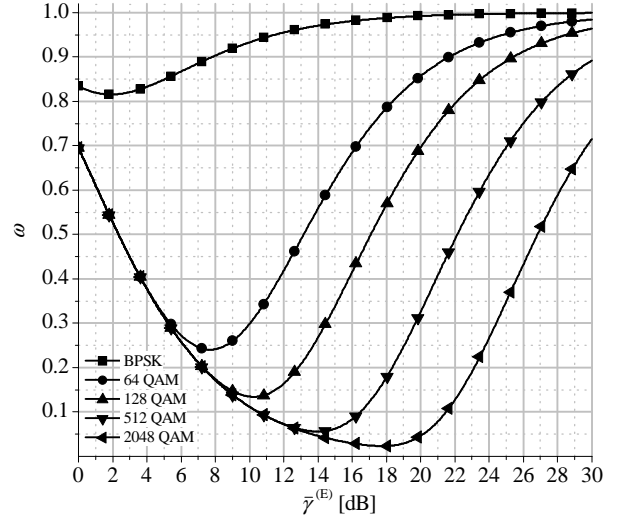


Fig. 4. Eve's outage probability  $\omega$  as a function of Eve's average SNR per bit  $\bar{\gamma}^{(E)}$  for an UEP LDPC coded transmission with BPSK-modulated public information bits and several QAM formats with Yarg labeling on the secret information bits.

as well as the value of  $\bar{\gamma}_{\text{opt}}^{(E)}$  for which  $\omega = \omega_{\min}$ . These values can be found according to the procedure described in Section III-B, and are also reported in Table I. Concerning Bob, we have fixed a maximum outage probability  $\eta_{\max} = \omega_{\min}$ , and computed the corresponding minimum value of his average SNR per bit,  $\bar{\gamma}_{\min}^{(B)}$ , according to (6). The values of  $\bar{\gamma}_{\min}^{(B)}$ , so obtained, are also reported in Table I.

Based on these results, we observe that, when both the public and the secret information bits are modulated with BPSK, the outage probability for Eve is always very large (more than 0.8). Therefore, although the system is theoretically feasible, in practice the fading nature of the channel rarely allows to achieve a successful transmission. The situation improves by adopting higher modulation orders for the private

information bits, which also increases the values of  $\alpha_s$ . This way, the outage probability for Eve is progressively reduced. When we adopt a QAM scheme with 2048 symbols, Eve's outage probability can be reduced down to 0.02. Under the hypothesis that Bob's outage probability is the same as Eve's outage probability (or less), we observe that there is a tradeoff between the outage probability and the security gap. In fact, if we are able to tolerate a high probability of outage, the system requires small security gaps (in the order of 10 dBs or even less). Instead, if we aim at small outage probabilities, we need large security gaps (in the order of 20 or 30 dBs).

On the other hand, high values of the outage probability (which mean that the system is often in outage) may be not much appealing in practice. However, as often occurs in physical layer security, the proposed coding and modulation scheme is able to offer a basic level of reliability and security, which can be then exploited by higher layer protocols and algorithms to reach the desired performance. Some examples of higher layer techniques of this kind are automatic repeat request protocols, to improve the transmission reliability, and AONTs acting on groups of concatenated blocks, to improve the transmission security without the need of any shared secret.

## VI. CONCLUSION

We have studied the BCC with quasi-static fading from a practical standpoint, by using the decoding error probability as a metric. We have proposed some practical coding and modulation schemes which can achieve some fixed reliability and security targets over this channel.

We have computed closed form expressions for the probability of outage at Bob's and Eve's, and assessed the security gap which is needed between their channels under the hypothesis of average CSI.

Our results show that high order modulation schemes are advisable for the secret information bits in order to achieve reasonably low values of the outage probability, although this yields some increase in the security gap.

As anticipated in Section II-A, future works will involve the assessment of the performance achievable in terms of the equivocation rate at Eve's, and the optimization of the coding and modulation scheme to work at a secret rate which approaches the equivocation rate.

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